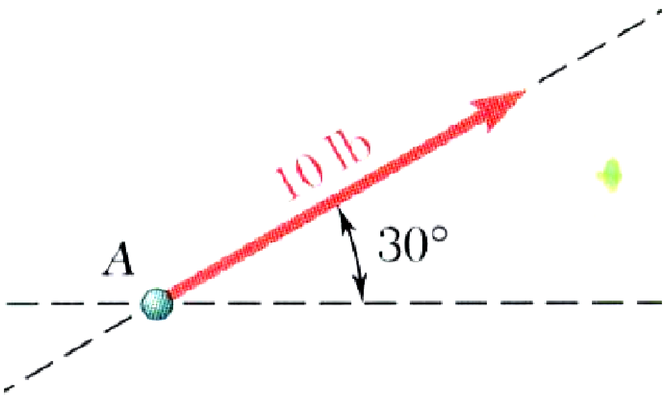
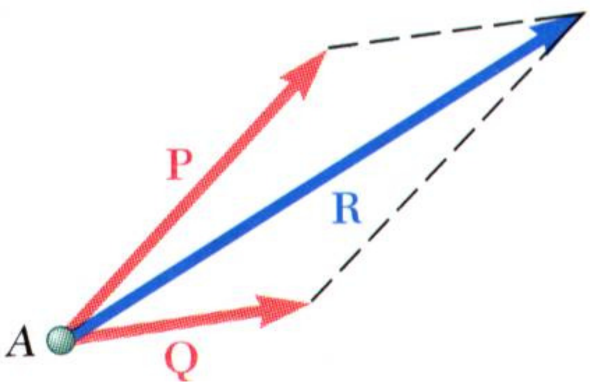
Introduction –Equilibrium of a Particle

The focus on *particles* does not imply a restriction to miniscule bodies. Rather, the study is restricted to analyses in which the size and shape of the bodies is not significant so that all forces may be assumed to be applied at a single point.

Resultant of Two Forces



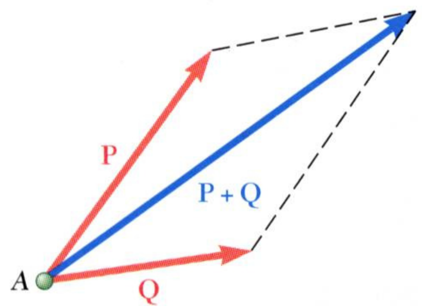
- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.

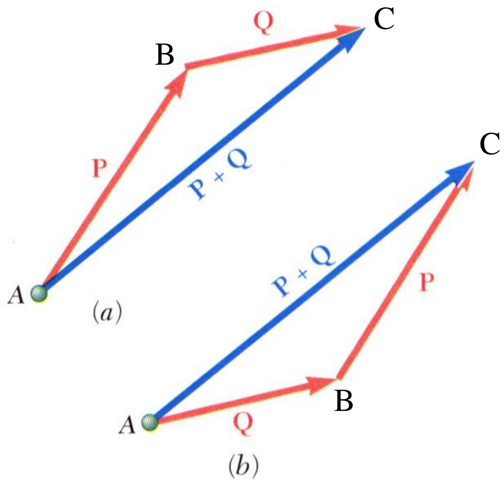


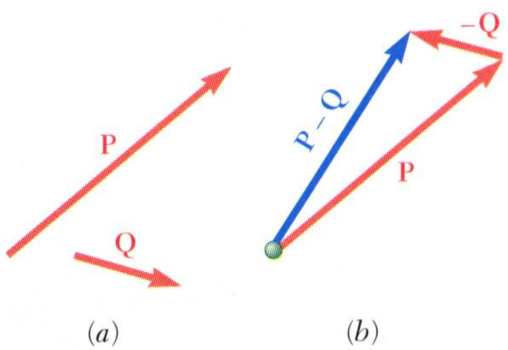
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.

- Force is a *vector* quantity.

Addition of Vectors







- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

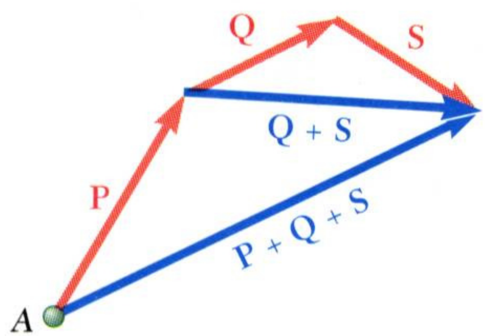
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative,

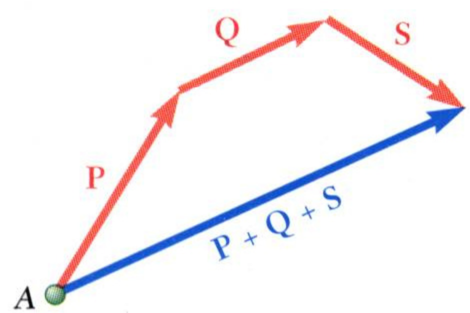
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Addition of Vectors

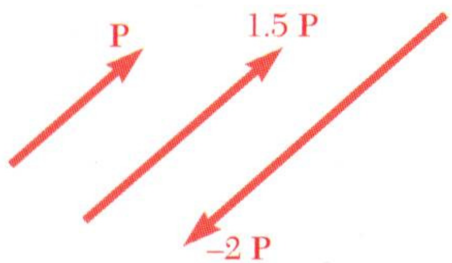


- Addition of three or more vectors through repeated application of the triangle rule



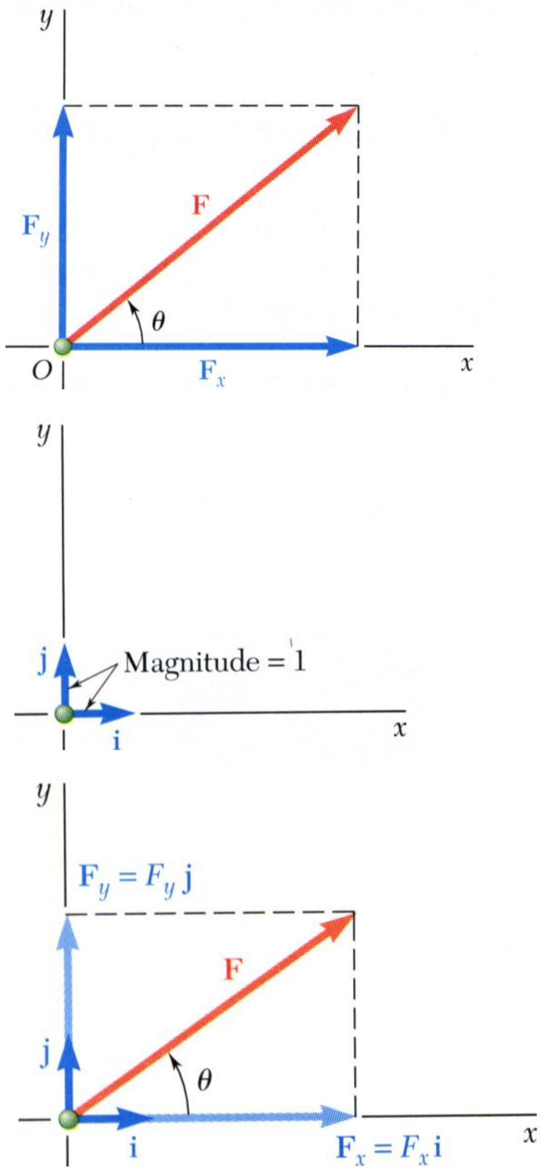
- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

$$\vec{P}+\vec{Q}+\vec{S}=\left(\vec{P}+\vec{Q}\right)+\vec{S}=\vec{P}+\left(\vec{Q}+\vec{S}\right)$$



- Multiplication of a vector by a scalar

Rectangular Components of a Force: Unit Vectors



- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

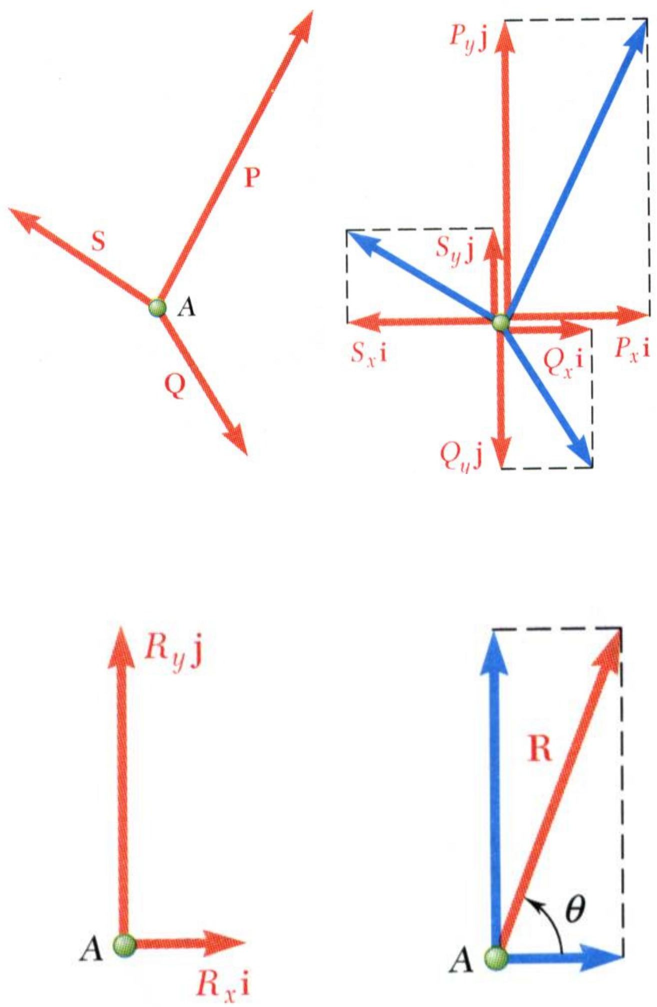
- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

Addition of Forces by Summing Components



- Wish to find the resultant of 3 or more concurrent forces,

$$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

$$\begin{aligned} R_x\vec{i} + R_y\vec{j} &= P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j} \\ &= (P_x + Q_x + S_x)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j} \end{aligned}$$

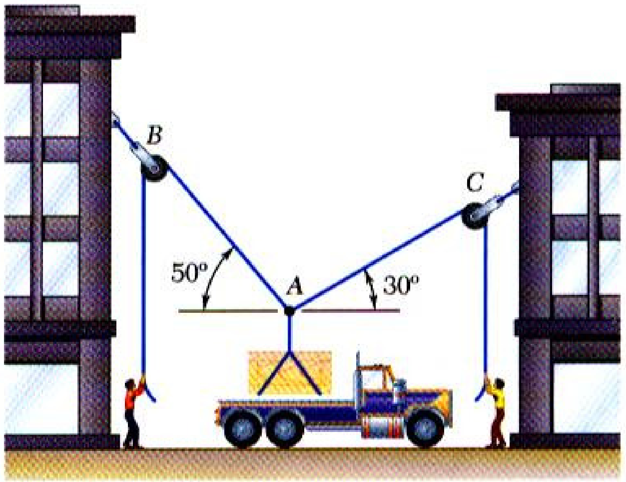
- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

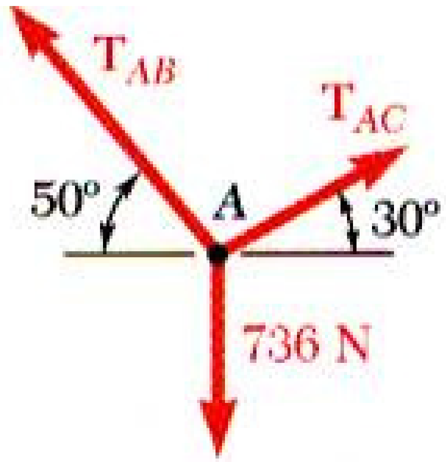
$$\begin{aligned} R_x &= P_x + Q_x + S_x \\ &= \sum F_x \end{aligned} \qquad \begin{aligned} R_y &= P_y + Q_y + S_y \\ &= \sum F_y \end{aligned}$$

- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Free-Body Diagrams

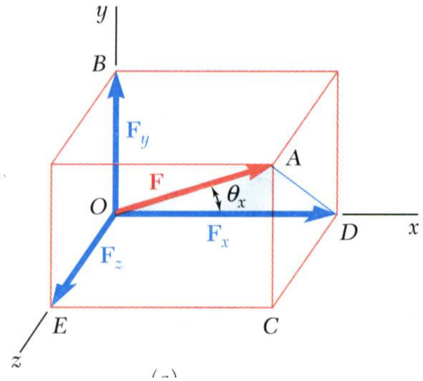


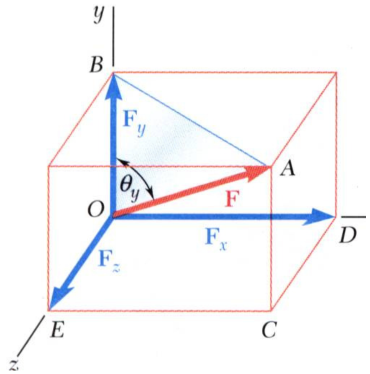


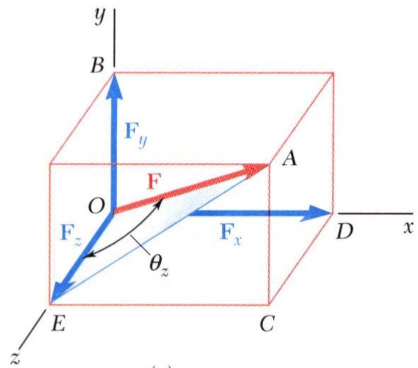
Space Diagram: A sketch showing the physical conditions of the problem.

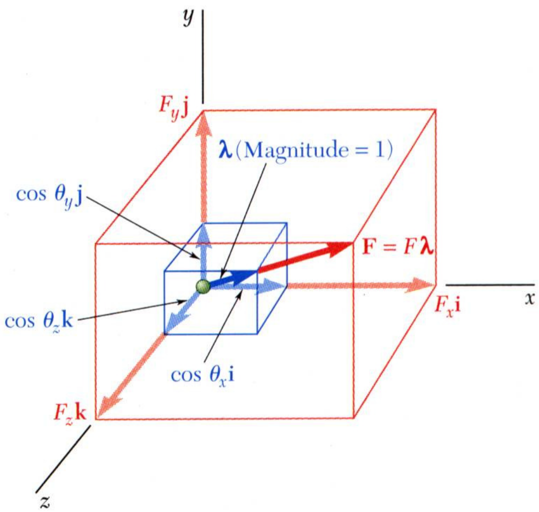
Free-Body Diagram: A sketch showing only the forces on the selected particle.

Rectangular Components in Space









- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\begin{aligned}\vec{F} &= F_x\vec{i} + F_y\vec{j} + F_z\vec{k} \\ &= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right) \\ &= F\vec{\lambda}\end{aligned}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$